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Creation of equations of movement of material particle on a plane inclined to vertical spinning axis

Goal. Output the equation of motion of a material particle on a rough plane, set at an angle to the vertical axis of its rotation. Methods. A general equation of motion of a particle in a plane inclined to the vertical axis of rotation is obtained, using the equation of the second kind of Lagrange, and the equation of transition to a moving coordinate system, which rotates along with the plane, is used. Results The physical model of motion of a particle is developed and the equation of motion of a material particle on a rough plane, set at an angle to the vertical axis of its rotation, is derived. Conclusions The obtained equations are transformed into known equations of motion of a particle along a rough horizontal plane, as well as on a rough vertical plane at vertical axes of their rotation, which confirms the correctness of the equations obtained for describing the movement of fertilizer particles along the lower shelf of the vanes of the centrifugal spreading of the tufts.

Key words: Lagrange equation of the second kind, moving coordinate system, rotation, reaction forces, friction, mass, acceleration, derivative, substitution, transformation, analysis.

Actuality of the question. The equation of motion of a particle in a plane inclined at an angle to the vertical axis of rotation is necessary to determine the motion of a material particle along the lower shelf of the blade of the general position of the centrifugal spreading of the tufts [1 - 3, 6 - 9].

Analysis of recent research and publications. An attempt to describe the movement of a material particle on a plane located at an angle to the vertical axis of its rotation was made long ago [2]. However, the references in [10] given in [2] without the derivation of the equation of motion of a particle on an ideally glanced plane are not perceived by the user and are not suitable for practical use. Therefore, this task is still unresolved.

The purpose of the research is to develop the method of withdrawal, the physical model of the motion of the particle, and deduce the equation of motion of the particle in a plane inclined at an angle to the vertical axis rotate and check the correctness of the results.

Research methods. The general equations of motion of a particle in a plane, set at an angle to the vertical axis of rotation, were obtained through the equation of the second kind of Lagrange, and the equation of transition to a moving coordinate system, which rotates with the plane, is used.

Research results. In contrast to the rotation of a plane surface or cone perpendicular to the axis of rotation, whose axis of symmetry coincides with the axis of rotation [7], the plane of the plane at an angle to the vertical axis of rotation changes as the parameters of motion of the particle and of the dynamics of the forces acting on her.

To avoid complications when determining the angles of inclination of the vectors of the relative and absolute velocities of the particle motion to the corresponding coordinate axes, a moving Cartesian coordinate system should be selected, The axis of OH lies in this very plane and is tightly connected with it. In it the definition of these angles along with the determination of the angles of inclination of the normal reaction of the surface N and the friction force fN to the coordinate axes is greatly simplified. For an inertial non-stationary Cartesian coordinate system, the Lagrange equation of the second kind has the form [2]:

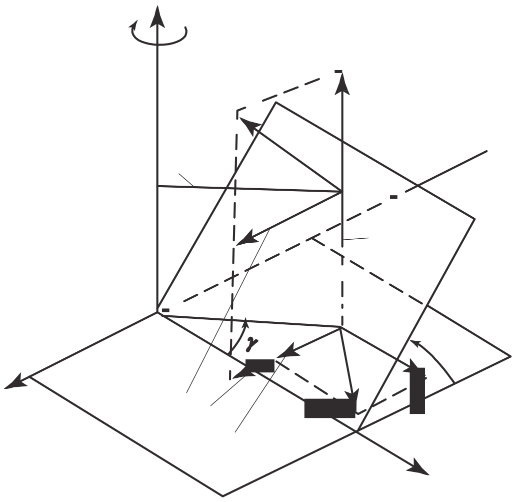
(1)

where Qx1, Qy1, Qz1 are live [2], or generalized [3, 5] forces that act on the axes X1, Y1, Z1.

The derivation of the derivatives in (1) gives the system of Newton's equations:

(2)

which correspond to the principle of independence of movements. The angles of inclination of the normal reaction of the plane N and the friction force fN to the axes of coordinates, which are active forces in the particle accelerating there, are shown in the figure. The point is that the plane itself under the particle rotates with an angular velocity ω (1 / s). Therefore, the transition To the coordinate system that rotates, there is also an objectively necessary one. Let the axis OX lie on the intersection of the plane with the horizontal plane, OY is horizontally perpendicular to the OX axis, and OZ - vertically along the axis of plane rotation. The particle travels on a plane at a point whose radius relative to the axis of rotation is R = x2 + y2. It lies at an angle γ to the XOZ plane and the OX axis (figure). Then the horizontal velocity of the point on the plane under the particle, which is equal to the portable velocity of the particle Vφ = Rω, gives on the X-axis the constituent X V R sin φ = ω γ, and OY - Y V R cos φ = ω γ. At the same time, the particle moves in a fixed space with velocities, which, by oasisx, respectively, is equal to AX AY Az V, V, V. Then the particle velocity relative to the plane on these axes is equal to: X AX V = -Rωsinγ + V; Y AY V = -Rωcos γ + V; Z AZ V = V Since it moves along the plane, then Z = -Ytgβ; Z Y V = -V tgβ; VZ VY tg • • = - β. It can be seen from the picture that tgγ = -y / x.



Z

$$x_1 = x\cos\phi - y\sin\phi\,; \quad y_1 = x\sin\phi + y\cos\phi\,; \qquad (3)$$
$$z_1 = z.$$

Similarly for dynamic parameters [3]:

$$Q_{X1} = Q_X\cos\phi - Q_Y\sin\phi;$$
$$Q_{Y1} = Q_X\sin\phi + Q_Y\cos\phi; \qquad (4)$$
$$Q_{Z1} = Q_Z,$$

where φ = ωt - the angle of rotation of the plane in the space, the radian; ω - angular velocity of rotation of a plane in space, 1 / s; X Y Z Q, Q, Q - meanings of generalized or live forces in the moving coordinate system; x, y, z is the value of the corresponding point coordinates in the moving coordinate system.

Twice differentiating the expressions (3) for x1 and y1 in time t, we obtain:

$$x_1 = x\cos\phi - x\omega\sin\phi - y\sin\phi - y\omega\cos\phi\,;$$
$$x_1 = x\cos\phi - x\,\omega\sin\phi - x\,\omega\sin\phi -$$
$$- x\,\omega^2\cos\phi - y\sin\phi - y\,\omega\cos\phi -$$
$$- y\,\omega\cos\phi + y\omega^2\sin\phi;$$
$$y_1 = x\sin\phi + x\omega\cos\phi + y\cos\phi - y\omega\sin\phi\,;$$
$$y_1 = x\sin\phi + x\,\omega\cos\phi + x\,\omega\cos\phi -$$
$$- x\omega^2\sin\phi + y\cos\phi - y\,\omega\sin\phi -$$
$$- y\,\omega\sin\phi - y\omega^2\cos\phi.$$

Summarizing the expressions before sin φ and cos φ with (4) gives:

$$x = (x - x\omega^2 - 2\,y\,\omega)\cos\phi - (y - y\omega^2 + 2\,x\,\omega)\sin\phi = (1/m)[Q_X\cos\phi - Q_Y\sin\phi];$$
$$y = (x - x\omega^2 - 2\,y\,\omega)\sin\phi + (y - y\omega^2 + 2\,x\,\omega)\cos\phi = (1/m)[Q_X\sin\phi + Q_Y\cos\phi]\,. \quad (5)$$

By solving (5) with respect to X (1 / m) Q and Y (1 / m) Q, we obtain:

$(1/m)Q = x - x\omega^2 - 2y\omega$
$(1/m)Q = y - y\omega^2 + 2x\omega$.   (6)

Then from the drawing we will write:

(7)

After the substitution (7) in (6) we have the equation of motion of a particle in a moving XYZ coordinate system in the form:

(8)

Consequently, we have 4 equations with 4 unknowns, namely x, y, z and N, of which we can easily find:

(9)

The substitution (9) in (8) gives the equation of motion of a material particle along a rough plane, set at an angle to the revolving axis:

It should be noted that the initial conditions in the resulting system of equations will be the initial values of the relative velocities of the particle motion relative to the plane, the definitions of which were given above. The singularity (9) is that for $\beta = 0$, N = g, and (8) goes into the system of equations obtained in [7] for a horizontal disk with a vertical axis of rotation. For $\beta = \pi / 2 = 900$; $\sin\beta = 1$; $\cos\beta = 0$; y = 0; N 2 x • = ω, and (8) is converted into a system of equations of motion of a particle along the vertical lateral margin of a radial blade with a vertical axis of rotation, which are given in [6]. For f = 0, the resulting equations are transferred to the system given for this case in [2]. All this once again confirms the correctness of the methods of making equations and their suitability for corresponding practical calculations.

When reversing the plane or its position on the other side of the OZ axis, as shown in the figure, the dependencies (3) and (4) will look like:

$x_1 = x\cos\phi + y\sin\phi$;
$Q_{x1} = Q_x\cos\phi + Q_y\sin\phi$;   (10)
$y_1 = y\cos\phi - x\sin\phi$;
$Q_{y1} = Q_y\cos\phi - Q_x\sin\phi$.

After two differentiations of the equations x1 and y1 in time t and the summation of the members before sin φ and cod φ, taking into account (1) and (10), we obtain:

$(x - x\omega^2 + 2y\omega)\cos\phi + (y - y\omega^2 - 2x\omega)\sin\phi = (1/m)(Q_x\cos\phi + Q_y\sin\phi)$;
$(y - y\omega^2 - 2x\omega)\cos\phi - (x - x\omega^2 + 2y\omega)\sin\phi = (1/m)(Q_y\cos\phi - Q_x\sin\phi)$.

When solving the last expressions for Qx and Qy, we find:

$x - x\omega^2 + 2y\omega = (1/m)Q$;
$y - y\omega \; - 2x\omega = (1/m)Q_y$; $z = (1/m)Q_z$.

Since the figure is meaningful after substituting these values in (11) we obtain:

…………………………..   (12)

These equations contain 4 unknowns: x, y, z and N. Multiplying the second equation by tg β, taking into account the 4th one, we have:

………………………………………….

adding to which left and right sides of the third equation, we obtain:

………………………………………..

from which we find:

……………………………………………

which differs from (9) signs. Its base in (12) gives an equation calculating system for this case. When N = 0; g (y 2 2 x) tg • = ω + ω β. In this case, the particle will not rest on the plane, but will fall freely under the action of gravity forces. At this time, the plane, escaping from the particle, gives it space for its free movement.

Conclusions

To obtain a system of equations of motion of a material particle on a rough surface, set at an angle to the axis of its revolution, it is necessary to move to a rotating Cartan coordinate system which rotates, taking into account the transformations (3, 4) and (10). The value of the accelerations of the material particle in this case should be determined through the double differentiation of expressions of type (3, 10). Generalized or living forces here are the normal reaction of this plane and the force of friction on it of the particle. The transition of the resulting equations of motion of a particle along this plane into the same equations for the horizontal plane as well as for the vertical plane with the vertical axis of rotation lying in this plane confirms the correctness of the methods of their derivation and their suitability for calculations the parameters of the movement of the fertilizer particle on the lower shelf of the centrifugal spreader blade.

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