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MATHEMATICAL MODELING AS A TOOL FOR RESEARCH AND DECISION SUPPORT IN LAND RECLAMATION

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Aim. The aim of the study is to substantiate the expediency and directions of applying mathematical modeling to solve research problems and to perform decision support in land reclamation.

Methods. Theoretical generalization and expert analysis.

Results. Regarding the climatic changes and the consequences of the military aggression of the Russian Federation there is an increased demand for the use of hydro-technical land reclamation (irrigation and drainage) to ensure the sustainability and efficiency of farming. This process requires the implementation of a large complex of studies on the substantiation of technical and technological parameters of irrigation and drainage systems at the stage of their design and the support of operative decision-making to ensure effective, environmentally safe operation of existing and newly created systems. Until recently, such studies were carried out mainly by establishing stationary experiments, which requires significant time, costs, and, moreover, gives answers only for the conditions, in which the experiments were conducted.

At the same time, the presence of a significant number of mathematical models that with sufficient adequacy describe various processes, which are studied experimentally, makes it possible to use them for modeling moisture and mass transfer in soils and, on this basis, more rigorously determine the parameters of irrigation and water regulation regimes. These problems for different methods of irrigation and water regulation can be solved on the base of solutions for direct problems using the Richards equation stated in terms of pressure in a one-dimensional (sprinkling, water regulation using drainage) or two-dimensional formulation (all varieties of drip irrigation).

Solving inverse problems using the same Richards equation makes it possible to determine the design parameters of subsurface drip irrigation and drainage systems.

An important class of problems that needs to be solved at the stages of designing irrigation and drainage systems are the problems of choosing crop rotation and water consumption rates. For the first time, the authors of the paper suggested that the optimal crop rotation should be selected using mathematical modeling methods.

Mathematical modeling is an almost irreplaceable tool in decision support systems for irrigation management or water regulation.

Conclusions. The restoration and development of irrigation and drainage systems in order to compensate for the losses caused to the agrarian sector of the economy by the military aggression of the Russian Federation (temporary loss of about 25% of agricultural land), as well as in the implementation of plans for the post-war reconstruction of Ukraine, should be based on the use of energy-, resource-saving, and environmentally safe systems of irrigation and drainage, the use of which will require significant amounts of research to substantiate their design parameters and operating regimes. It is proposed to use mathematical modeling methods as the main tool of these studies, including methods developed and tested by the authors of the paper.

Keywords: *mathematical modeling, irrigation and drainage systems, design and technological parameters, moisture transfer, evapotranspiration, water demand, water supply regimes.*

Introduction.

Climate change, especially its “hot” phase, which began in Ukraine in the late 1970s and early 1980s and is characterized by the highest rates of increase in the average annual air temperature in Europe (over 0.7°C/10 years) with practically unchanged precipitation, has led to an increase in the need for irrigation and water regulation [1]. This need is exacerbated by the consequences of the military aggression of the Russian Federation, primarily the destruction of the Kakhovka reservoir dam, as a result of which most of the irrigation systems in the southern regions of Ukraine were left without a water source, which will certainly cause their accelerated destruction and make it impossible to resume their operation after the restoration of the Kakhovka reservoir without taking measures to reconstruct and modernize them. The large volume of restoration work on the irrigation systems of the temporarily occupied territories, which will be carried out after their liberation, and the accelerated implementation of works to increase the area of irrigation and drainage, primarily in the Odessa region and Polissya, in order to strengthen the role of these regions in ensuring food security of Ukraine and the World, require the implementation of a significant amount of design work. These projects should provide for the use of the latest technologies and technical means of irrigation and water regulation, which, in turn, should be based on appropriate scientific justification. The effectiveness of management decisions in the operation of functioning irrigation and drainage systems also directly depends on the level of their scientific substantiation.

It is well-known that the main task of irrigation and drainage systems is to maintain the optimal soil water regime for the development of crops grown on irrigated and drained lands, because it is the level of soil moisture that is a critical factor influencing the processes in the “soil-atmosphere-plant” system and, accordingly, the productivity of cultivated crops. Therefore, assessments of the amount of soil moisture and its availability to plants is an important source of information for making the right decisions both at the design stage of irrigation and drainage systems and at the stage of their operation when choosing the crop to be grown and during operational irrigation management or water regulation.

Among the tasks that need to be solved at the stage of designing irrigation and drainage systems, there is also a problem of determining crop rotation and water requirements of agricultural crops that will be grown under the conditions of irrigation or water regulation. This information will become the basis for justifying water needs and calculating the parameters of irrigation or drainage systems.

Research purpose is to substantiate the feasibility and directions of applying mathematical modeling to solve research problems and support decision-making in land reclamation.

Materials and research methods.

The research was carried out using the methods of mathematical modeling, theoretical generalizations, and expert analysis.

Research results.

Models of the dynamics of moisture availability to plants. The simplest and most widely used models of the dynamics of moisture availability to plants (e.g. FAO-56 [2]) focus on the most accurate determination of evapotranspiration without taking into account the peculiarities of the temporal and spatial distribution of soil moisture pressures, which determine its availability to plants. Such models are effective in the case of continuously sown crops under the conditions of formation of homogeneously moistened zones, e.g. during sprinkler irrigation. In this case, the processes in the “soil-plant-atmosphere” system can be considered with sufficient accuracy in a one-dimensional approximation, and the hydro-physical properties of soils can be used to substantiate irrigation regimes.

Determining the distribution of moisture in the soil becomes fundamental in the case of significant soil heterogeneity, which can be formed either naturally or as a result of agricultural activities, or in the case of the use of water-saving irrigation methods, the purpose of which is to supply moisture precisely to those areas where it is needed and should be available to plants.

The simplest in this case are empirical models obtained, in particular, from experimental studies of moistened zones during drip irrigation [3,4]. However, taking into account significant differences in the structure and hydro-physical properties of soils even within one class, each specific empirical model has a very narrow range of applicability conditions.

The use of multilayer balance and differential models of moisture transport processes in soil becomes effective here. Currently, the most widely used for these purposes is the Richards differential equation [5], which describes moisture transport in soil media with scale-independent properties.

Considering the interdependence between pressure and the amount of soil moisture, which is described by water retention curves (WTC), the Richards equation can be stated both in terms of moisture content and in terms of pressure (water head). The second approach is more effective, since the use of such equations makes it possible to consider the aeration zone in one hydrodynamic scheme with groundwater horizons, and the confining bed of the aquifer can be used as a boundary condition in the corresponding boundary value problem. Also, discrete modeling in terms of water head has increased accuracy because at the moisture content value equal to the pre-irrigation threshold, small changes in it can lead to significant changes in pressure and, accordingly, in the availability of moisture to plants.

Thus, a model based on the Richards equation stated in terms of pressure in a two-dimensional formulation can have the following form [6]:

$$C(h)\frac{\partial H}{\partial t}=\frac{\partial}{\partial x}\left(k\left(H\right)\frac{\partial H}{\partial x}\right)+\frac{\partial}{\partial z}\left(k\left(H\right)\frac{\partial H}{\partial z}\right)-S, 0\le x\le L_x, 0\le z\le L_z, t\ge 0$$

where $h(x,z,t)=\dfrac{P(x,z,t)}{\rho g}$ is the water head, *m*, $H(x,z,t)=h(x,z,t)+z$ is the full moisture potential, *m*, $P(x,z,t)$ is the suction pressure, *Pa*, ρ is the water density, *kg/m*^{3}, g is the acceleration of gravity, *m/s*^{2}, $C(h)=\dfrac{\partial\theta}{\partial h}$ is the differential moisture capacity, $\theta(x,z,t)$ is the soil moisture content, *%*, $k(H)$ is the hydraulic conductivity, *m/s*, $S(x,z,t)$ is the source function, *1/s*, which models moisture extraction by plant roots and its delivery by subsurface drip irrigation.

At the lower boundary of the solution domain $z = L_z$ the condition $\frac{\partial H}{\partial z} = 0$ is set in the case of modeling the situation where irrigation does not have a significant impact on the movement of moisture and, at the appropriate depth, only its gravitational flow occurs. The condition $h = 0$ can be set for a known fixed groundwater level, and the no-flow condition $\frac{\partial h}{\partial z} = 0$ — in case of the confining bed.

At the upper boundary $z = 0$ a condition is set in the form $k(H)\frac{\partial H}{\partial z} = Q_e(t) - Q_p(t) - Q_i(x,t)$ where $Q_e(t)$, $Q_p(t)$, $Q_i(x,t)$ are the flows, *m/s*, caused, respectively, by evaporation, precipitation, and irrigation. On the lateral faces $x = 0, x = L_x$ a Neumann condition $\frac{\partial H}{\partial x} = 0$ is imposed. It can, in particular, describe the symmetry of processes in the zones of influence of neighboring drip pipelines.

To further improve the accuracy of the simulation, one can use more complex models, which include, in particular, double porosity models [7], models that take into account chemical processes [8], air phase transfer [9], or fractal properties of soils [10]. As for the latter, when modeling mass transfer processes in porous media of fractal structure, to which agricultural soils can be classified under certain conditions, fractional-differential equations can be applied [11]. The use of such models allows increasing the accuracy of modeling in complex geo-hydrological conditions by taking into account memory effects and non-local spatial interactions. Fractional-differential models can be considered as semi-empirical, where the orders of fractional derivatives are additional parameters, the values of which are identified in the process of adapting the model to specific crop growing conditions.

Let us note that the complexity of models leads to the need to accurately determine the values of their parameters, or to identify them based on a comparison of simulated and measured values.

Critical to the accuracy of modeling are the hydro-physical properties of soils — the forms of their WRC and the dependency of hydraulic conductivity on pressure. The sources of the forms of these dependencies can be, on the one hand, laboratory studies (see, e.g., [12]), or, on the other hand, models (e.g., Rosetta 3 [13]) based on statistical generalizations of laboratory studies data. Such models require a limited amount of input data — primarily, the data on soil texture. Since they generalize data on soil characteristics of geographically distant territories, the generated approximations often have a high level of variability. This necessitates local studies of the hydro-physical properties of soils both in the case of practical application and for scientific research. The studies conducted by the authors show that particular attention should be given to the most accurate determination of the filtration coefficient (the hydraulic conductivity of a water-saturated soil).

The second critical parameter, the definition of which should also be adapted to the actual conditions of crop cultivation, is the estimation of evapotranspiration. For such adaptation, approaches based on experimental determination of crop coefficients [14], a combination of several calculation methods [15], or the application of machine learning methods, in particular, neural networks [16], can be used. It should be noted that all these approaches require the collection of a large amount of monitoring data and are applicable only within one “crop-soil” pair.

The accuracy of modeling, which influences the effectiveness of management decisions and, accordingly, the final crop yield, significantly depends on the amount and accuracy of data on the state of the soil, the acquisition of which requires financial and time expenses. These data include laboratory studies of the hydro-physical properties of soils and the monitoring of the state of moisture supply to soils using micro-meteorological stations placed in the fields.

The modeling itself requires either the expenses for consulting services in complex cases or, in simpler cases, with the availability of freely or commercially available software (e.g. CropWat [17]), - the expenses for personnel training.

Decision-making methods based on forecast data.

Problems of modeling crop rotations and water consumption norms. Mathematical models of moisture transfer in one-dimensional approximation find their application in solving some important problems under conditions of large ranges of changes in the values of the initial data. In such situations, complicating the model, in particular with the transition to two-dimensional formulations, does not increase the accuracy of the modeling, since the potential ranges of changes in the initial data exceed modeling errors. Such problems include the problem of determining averaged irrigation rates.

The availability of estimates of irrigation rates averaged for typical crop growing conditions, formalized generalized yield models, and expert estimates of crop rotation efficiency allows providing recommendations on the choice of economically efficient crop rotations without conducting additional time-consuming vegetation experiments. One of the methods of such modeling, developed by the authors, is presented in [18].

Substantiation of the parameters of drip irrigation systems. As for medium- and long-term planning, especially in the case of water supply by drip irrigation systems or dual-action drainage systems, a wide range of problems arise related to expanding their application as one of the most effective methods of adapting agriculture to climate change.

Thus, the process of expanding the use of drip irrigation, especially its most promising variety - subsurface drip irrigation, will require solving a number of technical and technological problems related to substantiating the parameters of subsurface drip irrigation systems (SDI) depending on the conditions of their application and water supply regimes, the following of which allows achieving maximum economic effect with minimal capital expenses for the construction of SDI and specific irrigation water consumption per unit of yield of irrigated crops.

The design parameters of the SDI that need to be determined include the depth of installation and the distance between irrigation pipelines. Also, as experimental research data show, irrigation water consumption depends not only on the design parameters of the SDI, but also on the water supply regime. Thus, the type and optimal values of regime parameters also need to be determined.

Algorithms for automating the selection of SDI parameters and water supply regime can be considered as an optimization superstructure over moisture transfer models with objective functions based on economic efficiency assessment and constraints arising from the biological need to provide plants with moisture [19, 20]. A similar approach can be used for dual-action drainage systems [21].

To solve such problems, it is necessary to conduct scenario modeling of the entire growing season, or at least its part, within which plant moisture consumption is the greatest [19]. However, a model verified on the data of experimental determination of moistened zones can be used to determine the intervals of parameter values without conducting time-consuming laboratory studies of soil structure in specific fields.

Features of determining the parameters of the water supply regime. Let us note that additional difficulties in modeling are caused by the specifics of the development of the plant root system under point supply of irrigation moisture. In particular, the micro-pulse water supply regime for drip irrigation [22] provides, in contrast to continuous irrigation, the supply of water to the zones of maximum concentration of the plant root system in the shortest possible cycles of fixed or variable duration, adapting to the actual water demand within the daily cycle.

The use of micro-pulse irrigation regime makes it possible to form (in certain zones of the moistened volume) moisture levels higher than field capacity (FC). Thus, moisture and nutrients become more accessible to plants in the corresponding zones than in other irrigation regimes. This, in turn, leads to more intensive growth of the plant root system in the moistened zones [23]. In such a situation, the failure to model the dependency of the shape and size of the root system on moisture distribution can lead to the generation of physically non-consistent recommendations.

In the absence of specific experimental data on the dynamics of root system development of agricultural crops grown in Ukraine, the issue of their modeling and determining the sensitivity of moisture transport models to the accuracy of root system modeling remains open.

Operational irrigation management. In the case of operational irrigation management, the method developed by the authors [25, 26] was proved to be effective [24]. It forms a 5-day forecast of the need for irrigation based on publicly available forecast meteorological data with the adaptation of model parameters to the changes in crop growth conditions when a significant deviation of the simulated values of pressure from the measured ones is observed. This method is based on the modeling of moisture transport in a one-dimensional approximation and is applicable, primarily, for the case of sprinkler irrigation.

Since in drip irrigation the delay between the moment of instrumental determination of the need for irrigation and the water supply is minimal, the need to predict this moment is reduced and most often automatic control systems with sensors placed directly in the root systems of plants are used for operational management. The use of mathematical modeling becomes fundamental in this case to minimize the impact of measurement errors on the accuracy of irrigation scheduling [27], or in complex configurations of crop sowing and pipeline placement, in particular, the above-described case of the impact of this configuration on the development of the root system of plants.

Conclusions.

Mathematical modeling should be considered as an effective tool for solving a number of research problems in the development of irrigation and drainage systems designs, as well as in decision support systems used during their operation.

Particularly representative is the use of mathematical modeling usage to study moisture transfer processes in the vadoze zone of soils in order to substantiate the design parameters of irrigation and drainage systems along with the parameters of irrigation and water regulation regimes.

The use of mathematical modeling significantly expands the possibilities and increases the validity of calculations of evapotranspiration, irrigation schedules and rates, water supply and drainage regimes in decision support systems for irrigation management and water regulation.

Further research on expanding the areas of application of mathematical modeling in land reclamation should be aimed at establishing models of root system formation of various agricultural crops when they are grown under different methods and regimes of irrigation and water regulation; determining nutrient consumption coefficients when using the technologies for applying fertilizers and microelements with irrigation water, etc.

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